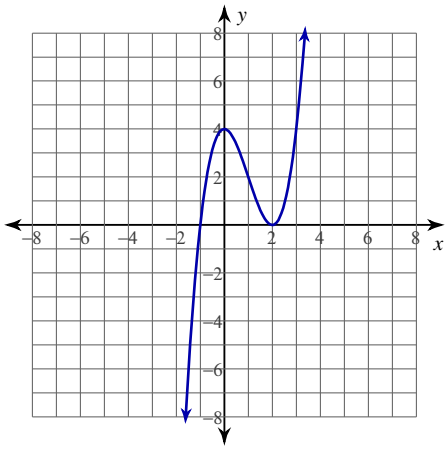
 Name_____

Intervals of Concavity

Date_____ Period____

For each problem, find the x-coordinates of all points of inflection, find all discontinuities, and find the open intervals where the function is concave up and concave down.

1) $y = x^3 - 3x^2 + 4$



2) $y = x^3 - 2x^2 - 2$

3) $y = x^4 + x^3 - 3x^2 + 1$

4) $y = \dfrac{1}{x - 3}$

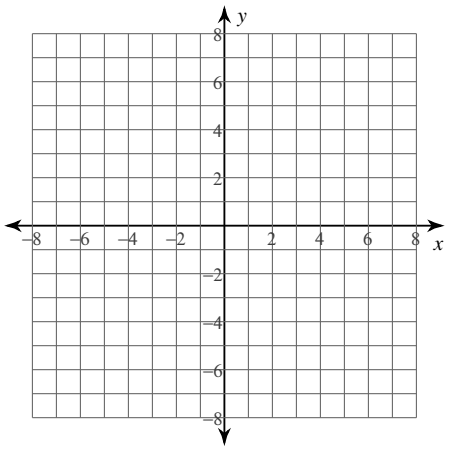
5) $y = -\dfrac{x^3}{x^2 - 4}$

6) $y = (5x + 30)^{\frac{2}{3}}$

7) $y = -\dfrac{3}{16}(x-1)^{\frac{4}{3}} - \dfrac{3}{2}(x-1)^{\frac{1}{3}} + 2$

Critical thinking question:

8) Sketch a continous curve $y = f(x)$ where $f(1) = 0$, $f'(0) = 0$, $f'(2) = 0$, $f''(x) < 0$ for $x < 1$, and $f''(x) > 0$ for $x > 1$.



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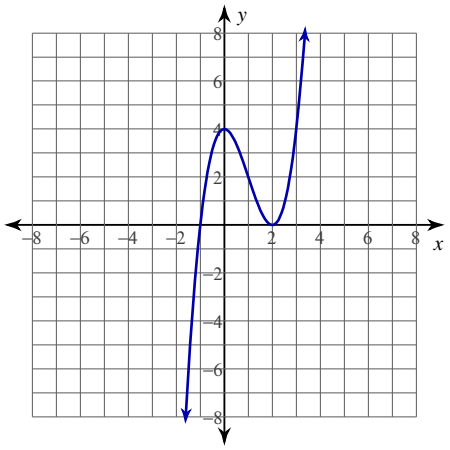
1) $y = x^3 - 3x^2 + 4$



Inflection point at: $x = 1$ No discontinuities exist.
Concave up: $(1, \infty)$ Concave down: $(-\infty, 1)$

2) $y = x^3 - 2x^2 - 2$

Inflection point at: $x = \frac{2}{3}$ No discontinuities exist.

Concave up: $\left(\frac{2}{3}, \infty\right)$ Concave down: $\left(-\infty, \frac{2}{3}\right)$

3) $y = x^4 + x^3 - 3x^2 + 1$

Inflection points at: $x = -1, \frac{1}{2}$ No discontinuities exist.

Concave up: $(-\infty, -1), \left(\frac{1}{2}, \infty\right)$ Concave down: $\left(-1, \frac{1}{2}\right)$

4) $y = \frac{1}{x - 3}$

No inflection points exist. Discontinuity at: $x = 3$
Concave up: $(3, \infty)$ Concave down: $(-\infty, 3)$

5) $y = -\dfrac{x^3}{x^2 - 4}$

Inflection point at: $x = 0$ Discontinuities at: $x = -2, 2$
Concave up: $(-\infty, -2), (0, 2)$ Concave down: $(-2, 0), (2, \infty)$

6) $y = (5x + 30)^{\frac{2}{3}}$

No inflection points exist. No discontinuities exist.
Concave up: No intervals exist. Concave down: $(-\infty, -6), (-6, \infty)$

7) $y = -\dfrac{3}{16}(x - 1)^{\frac{4}{3}} - \dfrac{3}{2}(x - 1)^{\frac{1}{3}} + 2$

Inflection points at: $x = 1, 5$ No discontinuities exist.
Concave up: $(1, 5)$ Concave down: $(-\infty, 1), (5, \infty)$

Critical thinking question:

8) Sketch a continous curve $y = f(x)$ where $f(1) = 0$, $f'(0) = 0$, $f'(2) = 0$, $f''(x) < 0$ for $x < 1$, and $f''(x) > 0$ for $x > 1$.